Exercise sheet 2

Computational Metaphysics

Benzmüller, Steen, Wisniewski Summer 2016

The exercises will be discussed in the tutorial session (wednesday 2pm).

Summary of so-far discussed natural deduction rules:

$$\frac{A \quad B}{A \wedge B}\ conjI \qquad \frac{A \wedge B}{A}\ conjunct1 \qquad \frac{A \wedge B}{B}\ conjunct2$$

$$\frac{A}{A \vee B}\ disjI1 \qquad \frac{B}{A \vee B}\ disjI2 \qquad \frac{A \vee B \quad \begin{array}{c}[A]\\ \vdots\\ C\end{array} \quad \begin{array}{c}[B]\\ \vdots\\ C\end{array}}{C}\ disjE$$

$$\frac{\begin{array}{c}[A]\\ \vdots\\ B\end{array}}{A \longrightarrow B}\ impI \qquad \frac{A \longrightarrow B \quad A}{B}\ mp \qquad \frac{\begin{array}{c}[A]\\ \vdots\\ \bot\end{array}}{\neg A}\ notI \qquad \frac{\neg A \quad A}{\bot}\ notE$$

$$\frac{\begin{array}{c}[\neg A]\\ \vdots\\ \bot\end{array}}{A}\ ccontr \qquad \frac{\neg\neg A}{A}\ notnotD \qquad \frac{}{\neg A \vee A}\ excluded\_Middle$$

Please solve all the following excercises using the Isabelle *system. Furthermore, start with the* `Example.thy` *(as explained in Excercise 1) and subsequently add your solutions to the remaining excercises as new subsections to this document. Finally, in excercise 4, we will create a fully verified .pdf file of your solutions.*

Exercise 1: Familiarizing with Isabelle.

As we will start working with the Isabelle system more often, this exercise helps you investigate the fundamental syntax and features of it.

(a) Start Isabelle[1]. The first start-up may take some time for technical reasons.

(b) Open the `Example.thy` theory[2] and explain its components. If you are not sure about certain keywords or contents try to speculate about its function.

(c) Recall the logical expressions stated in natural language from assignment 1. Please formalize your solutions (the resulting logical formulae) in Isabelle. The above `Example.thy` should have prepared you to write such terms and create necessary identifiers for individuals that are used in this terms. Create a new subsection in the `Example.thy` file and and your formalization there.

[1] Isabelle is installed on the pool computers. You can, of course, install it yourself on your machine. The software can be downloaded at `https://isabelle.in.tum.de/`. Additional configuration hints can be found at the lecture web site (under "Software-related resources", "Configuration hints")

[2] Download at lecture website, under "tutorial sessions", "assignments"

Exercise 2: Formalizing ND proofs.

Recall the ND proof tasks from exercise sheet 1:

(a) $A \wedge B \longrightarrow C$, $B \longrightarrow A$, $B \vdash C$

(b) $A \vdash B \longrightarrow A$

(c) $A \longrightarrow (B \longrightarrow C) \vdash B \longrightarrow (A \longrightarrow C)$

(d) $\neg A \vdash A \longrightarrow B$

(e) $\vdash A \vee \neg A$ (as before, the proof should not contain the rule *excluded_Middle*)

(f) $A \vee B$, $\neg A \vdash B$

(g) $\neg A \vee B \vdash A \longrightarrow B$

(h) $\vdash ((A \longrightarrow B) \longrightarrow A) \longrightarrow A$

(i) $A \leftrightarrow B \vdash (A \wedge B) \vee (\neg A \wedge \neg B)$

Formalize your proofs of the above statements from last week in Isabelle. You may want to consult the cheat sheet for the Isabelle ND proof templates and some examples[3]

Exercise 3: Formalizing Hilbert proofs.

For some apparent reason, philosophers love Hilbert systems (that is, a Hilbert-style proof system as introduced on exercise sheet 1) – we can do that as well. In this exercise we formalize a formal proof in such a propositional Hilbert-system in Isabelle. Since we are not going to stress Hilbert proofs in this lecture, we only want you to learn the fundamental differences to the above ND proofs in a practical example. To that end, please formalize a Hilber-style proof for $(A \longrightarrow B), (B \longrightarrow C) \vdash A \longrightarrow C$ in Isabelle.

Recall that in a Hilbert-style proof, only one inference rule, the modus ponens, is allowed. A prepared `Hilbert.thy` theory file (with a small example proof), containing the axiom schemes of the Hilbert system, can be downloaded at the lecture web site[4]. You may use this .thy file to solve this excercise (i.e. adding your solution to the downloaded `Hilbert.thy`).

Exercise 4: Creating a verified paper.

The above exercises were solved using the Isabelle system and therefore formalized inside one or more `.thy` files. We can now create a `.pdf` file containing your verified solutions and additional comments. Please create such a file (e.g. containing `Example.thy` and `Hilbert.thy` if you chose to add your solutions to that files) and upload it together with your theory file before the submission deadline. Hints on how to create such a `.pdf` file and how to inlcude one or more `.thy` files can be found at the lecture website[5]

[3]The cheat sheet can be found at the lecture web site, cf. section "tutorial sessions", "assignments"

[4]Cf. section "tutorial sessions", "assignments".

[5]Cf. Section "tutorial sessions", "assignments".