Exercise sheet 6

Computational Metaphysics

Benzmüller, Steen, Wisniewski Summer 2016

The exercises will be discussed in the tutorial session (wednesday 2pm).

Please solve all the following excercises using the Isabelle system. Add allyour solutions to the same `.thy` *file, create a* `.pdf` *file and upload both using the KVV system. The* `.thy` *file (andthus the theory) should benamed using the format* `Lastname`$_1$`Lastname`$_2$`Ex06.thy` *as in* `MüllerMeierEx06.thy`*.If you are using a temporary account, please also state the account name somewhere in yoursolution.* **You may use all proof tactics except for `smt` for solving this exercise sheet!**

Please note that since we are using the QML library, the type of individuals is now called μ. Also, when inside of modal logic formulae, the type of truth values is called σ.

Exercise 1: ***Modal Logic.***

As a soft start into modal logic we will have an intuitive introduction at the tutorial sessions. We will look into the semantics of modal logic later, where we will also investigate the embedding that is included using the QML library.

Modal logic tries to capture the meaning of necessity which is represented as a new connective $\Box$. There is no simple (and uniform) description of what "it is necessary that ..." formally means. Nevertheless, we want necessity to respect the following:

theorem K:
shows $\lfloor\Box(X \to Y) \to (\Box X \to \Box Y)\rfloor$

(a) Using this fact, we can derive simple tautologies such as: "If its not possible that all humans are born equal and some humans are not allowed to vote, and it holds necessarily that all humans are born equal, then it is necessary that all humans are allowed to vote.". Please formalize and prove this statement.

(b) Many philosophers argue that the above fact K does not properly characterize necessity. We can support this claim by trying to prove the following statement that should, intuitively, hold: "If the fact that it is raining necessarily implies that the street is wet, and it holds that it is raining, then it follows that the street is wet.".

Formalize this statement. Use nitpick to verify that we indeed cannot prove it.

(c) In order to allow the above statement to be a theorem, we include the axiom

axiomatization where
T: $\lfloor\Box A \to A\rfloor$

Show that you can now prove the above statement.

(d) Again, we might argue that the above axiomatization is still not strong enough, hence we want to include theorems of the form

axiomatization where
5 : $\lfloor\Diamond A \to \Box\Diamond A\rfloor$

Briefly comment on this axiom. Do you think it is a reasonable property of necessity?

Show that, if we assume 5, we can prove the statement "If it is raining, then is it necessarily possible that it is raining.".

Hence, we might want to include the axiom 5. We will see later on, that there are a lot of different axiomization schemes for modal logic which may be desirable in some contexts but may be inappropriate in other ones.

Exercise 2: ***First Modal Ontological Argument.***

Another version of the Ontological Argument we are looking at for this exercise is one of the first formalizations, that employs modern modal aspects. It was suggested by Norman Malcom (Philosophical Review, vol. 69(1), 1960, pp. 41–62) in order to address some flaws of the original version of Anselm. It is stated as follows (slightly modified):

(a) Being god is as a conceptual matter (that is, as a matter of definition) being unlimited.

(b) The existence of an unlimited being is either logically necessary or logically impossible.

(c) The existence of an unlimited being is not logically impossible.

(d) Therefore the existence of god is logically necessary.

Please formalize this argument using the modal logic embedding discussed in the tutorial session. Hint: Formalize statements (a)–(c) as axiomatization and the last statement (the conjecture (d)) as a theorem.

Give a proof that (d) is a consequence of (a) – (c). Do you need any axioms (such as T or 5) to prove it?

Exercise 3: ***Preparation for next week.***

In the next week we will have a closer look into Gödel's Ontological Argument and a more formal introduction into modal logic.

For good preparation, please read the SEP article about Modal logic(http://plato.stanford.edu/entries/logic-modal/) especially the chapters 1-2 (What is modal logic), 7 (Modal Axioms and Conditions on Frames) and 14 (Quantifiers in Modal Logic). Don't be afraid if you do not understand all the details. The upcoming lectures will clarify the topic and give a platform for your questions.