

The exercises will be discussed in the tutorial session (wednesday 2pm).

Please solve exercise 1 and 2 in a reasonable format of your choice (e.g. as textual proof, as a graph drawing, ...). Solve exercise 3 in Isabelle. Upload the `.thy` file, the compiled `.pdf` file and all electronic solutions for exercise 1 and 2 to the KVV system. The files should contain the names of all your group members.
Handwritten solution can be handed in at our office. **You may use all proof tactics except for `smt` for solving this exercise sheet!**

Exercise 1: Deduction Property.

In classical logic the Deduction Property

$$A, B \models C \quad \Leftrightarrow \quad A \models B \rightarrow C$$

is often used interchangeably. In modal logic, however, this property does not hold unconditionally any more.

Construct a counter model to disprove the property for global consequence. *Hint: Construct a model structure, in which A, B is not valid, but A is. Then use it to show that $B \rightarrow C$ is not valid in this model.*

Exercise 2: Frame Correspondence.

Prove, that for each frame \mathcal{A} the following conditions are equivalent

- (i) The relation R is transitive.
- (ii) For each formula φ , $\mathcal{A} \models \Box\varphi \rightarrow \Box\Box\varphi$.
- (iii) For some constant P , $\mathcal{A} \models \Box P \rightarrow \Box\Box P$.

Exercise 3: Local vs. Global Consequence.

We briefly heard in the last lecture, that there are two versions of consequence. After solving Exercise 1 you might have an idea, what the difference is. To get a further taste of these subtle differences, try to prove or refute the following statements in Isabelle:

- (a) **theorem** $[p] \Longrightarrow [\Box p]$
- (b) **theorem** $[p \rightarrow \Box p]$

Exercise 4: Identity and Modality.

Using equality in modal logic introduces a range of problems. A famous argument by Gottlob Frege goes as follows:

The morning star is the evening star.

In ancient times people knew, that the morning star is the morning star and the evening star is the evening star.

Hence, they knew that the morning star is the evening star.

This argument is of course ambiguous, since the knowledge both denote the planet Venus is a fairly young discovery.

To be prepared for the upcoming lectures and the group projects, please read the SEP article <http://plato.stanford.edu/entries/identity-transworld>.