Exercise sheet 9

Computational Metaphysics

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The exercises will be discussed in the tutorial session (wednesday 2pm).

Please solve all the following excercises using the Isabelle system. Add allyour solutions to the same archive (e.g. `.zip` file), create a `.pdf` file and upload both using the KVV system. The `.zip` file should be named using the format $Lastname_1Lastname_2Ex09.zip$ as in MüllerMeierEx09.zip.If you are using a temporary account, please also state the account name somewhere in yoursolution. **You may use all proof tactics except for `smt` for solving this exercise sheet!**

Exercise 1: Scott's variant of the Ontological Proof.

Scott's variant of the ontological proof has been formalized within the `Scott.thy`[1] theory. It imports the known `QML.thy` formalization of quantified modal logic and assumes a modal logic S5. Since we learned about the semantics of modal logic in the last sessions, we can – for this exercise – safely ignore the details of the embedding itself. Please focus on the details of the formalization of Scotts variant of the Ontological proof itself. Try to understand, how the embedding is used to express the steps of the argument.

Similar applications of modal logic (or other non-classical logics) will be employed during the group work phase on different topics. It is important to see how the underlying embedding of the expressive logic (here: quantified modal logic) allows us to express complex statements.

Exercise 2: Semantical Embedding I : Easier S5 Embedding.

The modal logic S5 restricts the accessibility relation R to be an equivalence relation. A common (not generally agreed on) alternative interpretation is that the $\Box$-operator ranges over all possible worlds, not just over those which are accessible (we call this modal logic S5U).

(a) Please, adjust the QML embedding `QML.thy` to an embedding `QMLS5U.thy` which formalizes modal logic S5U. You may of course copy-and-paste relevant parts of the regular QML embedding and adjust only the appropriate details.
(You are not allowed to assume or axiomatize the S5 axioms.)

(b) Check whether your embedding satisfies the S5 axioms (you can choose a concrete S5 axiomatizaion you want).

(c) Formulate the axioms of Scott's version of the Ontological Proof in a separate theory file which includes your `QMLS5U.thy`. If your new embedding is simple enough, you should be able to prove the conjecture $\Box\exists x.G(x)$ without any intermediate steps (Use sledgehammer to find the proof tactics).

(d) Discuss if you see any problem in defining modal logic S5 as we did in (a).

[1] See lecture website, "assignments" section, exercise 9, additional materials.

Exercise 3: Semantical Embedding II : Temporal Logic (TL).

Temporal logic deals with the validity of formulas over time. Two modal operators (and their dual forms) are introduced:

P "At some point in the past, it has been..."

F "At some point in the future, it will be ..."

H "(In the past), it has always been ..."

G "(In the future), it will always be ..."

The weakest form of temporal logic (K_t) can thereby defined by the following axiom scheme:

$G(\Psi \rightarrow \Phi) \rightarrow G\Psi \rightarrow G\Phi$	Axiom K for G
$H(\Psi \rightarrow \Phi) \rightarrow H\Psi \rightarrow H\Phi$	Axiom K for H
$\Psi \rightarrow GP\Psi$	Symmetry I
$\Psi \rightarrow HF\Psi$	Symmetry II

Classically (in western civilization) we assume time additionally to be transitive, never ending (in both future and past) and time behaves like a line (a timeline).

These properties are captured by the axioms

TRAN $G\Psi \rightarrow GG\Psi$, $H\Psi \rightarrow HH\Psi$

NOEND $G\Psi \rightarrow F\Psi$

NOBEG $H\Psi \rightarrow P\Psi$

LIN $((PF\Psi \vee FP\Psi) \rightarrow P\Psi \vee \Psi \vee F\Psi$

Please solve the following exercises.

(a) Come up with an embedding for the described temporal logic.[2] You may use the QML embedding as an inspiration. You may not assume or axiomatize any of the axioms above.

(b) Verify, that all the above axioms hold for your embedding.

(c) Model the following statement in the defined temporal logic and verify its validity:

If any entity is dead, it will be dead forever.
Eventually, every entity is dead.
Every entity is not dead at some point.
From this we can conclude, that every dead entity has not been dead[3] *at some point in the past.*

Optional question: Is the concept not dead captured by the concept of living in this model?

[2]You may read the article on temporal logic in the SEP `http://plato.stanford.edu/entries/logic-temporal/#PriBasTenLogTL`.

[3]The phrase "has not been" corresponds to "was never" for an arbitrary point in the past.